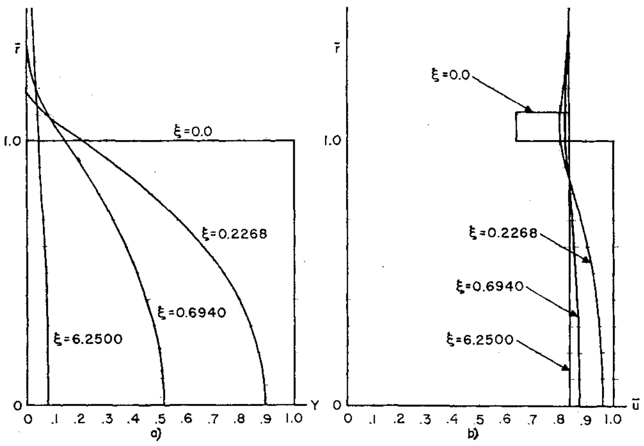


**Fig. 4 a) Radial mass concentration; b) radial velocity distribution**

by Alpinieri indicate that $\xi = 6.25$ corresponds to about 15 radii, and, consequently, it can be inferred that the classical eddy-viscosity law is not valid. If, on the other hand, the dynamic viscosity coefficient, as suggested by Ferri,[1, 3] is used,

$$\overline{\rho\epsilon} = kb_{1/2}[(\rho u)_{\max} - (\rho u)_{\min}] \tag{10}$$

$\xi = 6.25$ corresponds now to about 15 radii, which is in agreement with the experimental results. Hence, it appears that the viscosity law described by Eq. (10) is a better formulation for describing the physical phenomena.


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# Minimizing Weight Penalty for VTOL Performance


B. Saelman*

*Lockheed California Company, Burbank, Calif.*


GENERALLY, for a given mission, in comparing VTOL aircraft with conventional aircraft, the weight increment in the powerplant, rotor groups, and required fuel associated with lifting systems will be offset partially by reduced weight of landing gears, wing, and possibly the tail. The wing of the VTOL machine might be smaller in some instances, since its geometry is no longer dependent on conventional take-off and landing; the landing gear need not be designed for high-speed rolling take-offs and landings, and the tail need not be sized by take-off and landing requirements or high-speed requirements. To formulate the problem mathematically, proceed from the growth factor concept:

$$\partial W_G / \partial W_F = K \tag{1}$$

The fixed weight includes payload, crew, crew provisions, passengers and provisions, and all other weights that are not directly dependent upon airplane size. The growth factor thus is defined as the total gross weight increment required to accommodate a unit increment of fixed weight or, more generally, the total gross-weight increment corresponding to a unit initial-weight increment of any nature. Although the growth factor expressed by Eq. (1) generally is not constant with gross-weight variations, since it could reduce, for example, with increased structural efficiency as size increases, it will be assumed to be constant for this discussion. Under this assumption, one can write

$$\frac{W_{F_1}}{W_{G_1}} = \frac{W_{F_1} + \Delta W_E + \Delta W_{\text{FLAPS}} + W_R + \Delta W_{\text{FUEL}} - \Delta W_W - \Delta W_T - \Delta W_{LG}}{W_{G_2}} \tag{2}$$

and

$$R = \frac{W_{G_2}}{W_{G_1}} = \frac{\dfrac{W_{F_1}}{W_{G_1}} + \dfrac{\Delta W_E}{W_{G_1}} + \dfrac{\Delta W_{\text{FLAPS}}}{W_{G_1}} + \dfrac{W_R}{W_{G_1}} + \dfrac{\Delta W_{\text{FUEL}}}{W_{G_1}} - \dfrac{\Delta W_W}{W_{G_1}} - \dfrac{\Delta W_T}{W_{G_1}} - \dfrac{\Delta W_{LG}}{W_{G_1}}}{W_{F_1}/W_{G_1}} \tag{3}$$

The gross-weight increment for VTOL operation will be minimized when the ratio given by Eq. (3) has its least value. With $W_{G_1}$ fixed, this can be achieved by minimizing $W_{G_2}$. The ratio $R$ is a minimum when

$$\frac{dR}{dW_{G_1}} = \frac{1}{c} \sum \frac{d(W_i/W_{G_1})}{dW_{G_1}} = 0 \tag{4}$$

From Eq. (3), $R$ vs $W_{G_1}$ can be plotted and the minimum value of $R$ and the corresponding value of $W_{G_1}$ obtained. However, this procedure implies that the optimum gross weight for conventional performance first must be selected and then the payload matched to this gross weight; this is the converse of the usual procedure. However, it still may indicate one possible optimum design point.

Another approach is to require high performance for the conventional aircraft, such as high speed, operating altitude, rate of climb, etc., so that the powerplant size and weight differential between the conventional and VTOL aircraft are minimized.

The gross-weight ratio $R$ can vary from about 1.1 for a supersonic transport to about 2.0 for a modern fighter. The reason that the fighter gross-weight ratio is so high is that the weight increments in the powerplant and fuel required for VTOL performance are not offset significantly by reductions in other components. Thus, for a VTOL fighter using lift engines, the gross-weight increment could be minimized by reducing the specific weight and specific fuel consumption of the engines.

It is obvious from Eq. (3) that the VTOL weight increment reduces with the growth factor. Reduction in growth factor can be accomplished by the many weight-reduction devices that have been discussed. It is interesting to note that for some engines the specific weight will increase and the specific fuel consumption will decrease with size or thrust in some ranges, indicating that this is an area for optimization.

Received by IAS November 26, 1962.

* Design Specialist.